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## FAST TRACK COMMUNICATION

# Thermal Casimir–Polder force between an atom and a dielectric plate: thermodynamics and experiment

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Online at [stacks.iop.org/JPhysA/41/432001](http://stacks.iop.org/JPhysA/41/432001)**Abstract**

The low-temperature behavior of the Casimir–Polder free energy and entropy for an atom near a dielectric plate are found on the basis of the Lifshitz theory. The obtained results are shown to be thermodynamically consistent if the dc conductivity of the plate material is disregarded. With inclusion of dc conductivity, both the standard Lifshitz theory (for all dielectrics) and its generalization taking into account screening effects (for a wide range of dielectrics) violate the Nernst heat theorem. The inclusion of the screening effects is also shown to be inconsistent with experimental data of Casimir force measurements. The physical reasons for this inconsistency are elucidated.

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**1. Introduction**

Recently, considerable interest has been focused on the interaction of atoms with metal and dielectric plates (walls) at separation distances  $a$  from about 100 nm to a few micrometers where the retardation effects of the electromagnetic field play an important role. The relativistic description of the fluctuating interaction of atoms with an ideal metal plate at zero temperature was pioneered by Casimir and Polder [1] who obtained the interaction energy in the form  $E(a) = -3\hbar c\alpha_0/(8\pi a^4)$ , where  $\alpha_0 \equiv \alpha(0)$  is the static atomic polarizability,  $\hbar$  and  $c$  are the Planck constant and the velocity of light respectively. Lifshitz [2] developed the general theory of dispersion forces between two dielectric semispaces at a temperature  $T$  in thermal equilibrium with plane parallel boundary surfaces separated by a gap  $a$  of arbitrary width much larger than interatomic distances. This theory describes material properties with a dielectric permittivity that depends only on frequency. It includes both nonrelativistic (when  $a \ll \lambda_0$  where  $\lambda_0$  is the characteristic absorption wavelength of the semispace material) and relativistic (when  $a \gg \lambda_0$ ) limiting cases. If the material of one of the plates is rarefied, the general formula for the energy of the atom–wall interaction is obtained. For an ideal

metal plate at  $T = 0$  it leads to the Casimir–Polder result. In the high-temperature (large separation) limit the interaction free energy of an atom with an ideal metal plate is given by  $\mathcal{F}(a, T) = -k_B T \alpha_0 / (4a^3)$ , where  $k_B$  is the Boltzmann constant. For a dielectric plate with the static dielectric permittivity  $\epsilon_0 \equiv \epsilon(0)$ , the free energy acquires an additional factor  $r_0 = (\epsilon_0 - 1) / (\epsilon_0 + 1)$  which goes to unity when  $\epsilon_0 \rightarrow \infty$ .

In recent years the Casimir–Polder and Lifshitz formulae have been used for the interpretation of many experiments on quantum reflection and Bose–Einstein condensation of ultra-cold atoms near a variety of surfaces (see, e.g., [3–7] and references therein). The Lifshitz theory was also extensively applied for the interpretation of measurements of the Casimir force between two macrobodies made of metals and semiconductors (see, e.g., [8–18]). All these experiments have attracted widespread attention from diverse fields ranging from condensed matter physics and nanotechnology to atomic physics, elementary particle physics, gravitation and cosmology. However, the application of the Lifshitz theory to the real material bodies used in experiments has resulted in a puzzle. It was found that for metallic test bodies with perfect crystal lattices the calculational results are in contradiction with thermodynamics [19, 20] and experimental data [10–12] if the relaxation processes of conduction electrons are included into the model of the dielectric response used in the Lifshitz theory. For semiconductor and dielectric materials whose conductivity goes to zero with vanishing temperature, the calculational results using the Lifshitz theory were also shown to be in contradiction with thermodynamics [21–23] and experiment [16, 17, 24] when the dc conductivity of a dielectric or a high-resistivity semiconductor plate is included.

An interesting attempt to find the physical explanation for some of these puzzling results is undertaken in [25]. It takes into account the Debye screening of the electrostatic field by free charge carriers in the conductor leading to a modified reflection coefficient for the transverse magnetic (TM) mode at zero frequency in comparison to the Lifshitz theory (the above-mentioned puzzles arise only in the zero-frequency term of the Lifshitz formula). The reflection coefficient for the transverse electric (TE) mode at zero frequency for dielectric materials is equal to zero regardless of the dc conductivity.

In this communication we find the low-temperature behavior of the entropy of the Casimir–Polder atom–plate interaction both in the framework of the commonly accepted Lifshitz theory and its generalization taking into account the screening effects. We demonstrate that if the dc conductivity of dielectrics is disregarded the Casimir–Polder entropy goes to zero when the temperature vanishes, i.e., the Nernst heat theorem is satisfied. Thus, we show that in this case the standard Lifshitz theory of atom–wall interaction is in agreement with thermodynamics. If the dc conductivity is included, the standard Lifshitz theory violates Nernst’s theorem. The generalization [25] is in agreement with Nernst’s theorem for dielectrics whose charge carrier density vanishes when  $T$  goes to zero. However, for dielectrics whose charge carrier density is temperature-independent (for such materials conductivity goes to zero with  $T$  not due to the vanishing  $n$  but due to the vanishing mobility of the charge carriers) the generalization of the Lifshitz theory taking into account the screening effects is shown to violate the Nernst theorem. Finally we demonstrate that the suggested generalization of TM reflection coefficient at zero frequency is inconsistent with the measurement data of the difference Casimir force between a metal sphere and a semiconductor plate [16, 17].

## 2. Low-temperature behavior of the Casimir–Polder entropy

We start with the standard Lifshitz formula for the free energy of an atom–plate interaction [2, 4, 5]

$$\mathcal{F}(a, T) = \frac{k_B T}{8a^3} \sum_{l=0}^{\infty} \prime \Phi_A(\zeta_l), \tag{1}$$

where  $\zeta_l = \xi_l/\omega_c$ ,  $\xi_l = 2\pi k_B T l/\hbar$  are the Matsubara frequencies,  $\omega_c = c/(2a)$ , the prime indicates that the  $l = 0$  term has to be multiplied by 1/2, and

$$\Phi_A(x) = -\alpha(i\omega_c x) \int_x^{\infty} dy e^{-y} \{2y^2 r_{TM}(ix, y) - x^2[r_{TM}(ix, y) + r_{TE}(ix, y)]\}. \tag{2}$$

The reflection coefficients are defined through the frequency-dependent permittivity  $\varepsilon \equiv \varepsilon(i\omega_c x)$

$$r_{TM}(ix, y) = \frac{\varepsilon y - \sqrt{y^2 + x^2(\varepsilon - 1)}}{\varepsilon y + \sqrt{y^2 + x^2(\varepsilon - 1)}}, \quad r_{TE}(ix, y) = \frac{y - \sqrt{y^2 + x^2(\varepsilon - 1)}}{y + \sqrt{y^2 + x^2(\varepsilon - 1)}}. \tag{3}$$

The atomic dynamic polarizability can be represented with sufficient precision using the single-oscillator model [4]

$$\alpha(i\omega_c \zeta_l) = \frac{\alpha_0}{1 + \beta_A^2 \zeta_l^2} \tag{4}$$

with a dimensionless constant  $\beta_A$ .

Using the Abel–Plana formula [26], the free energy (1) can be equivalently represented in the form

$$\mathcal{F}(a, T) = E(a) + i \frac{k_B T}{8a^3} \int_0^{\infty} dt \frac{\Phi_A(i\tau t) - \Phi_A(-i\tau t)}{e^{2\pi t} - 1}, \tag{5}$$

where  $E(a)$  is the Casimir–Polder energy at zero temperature,  $\tau = 2\pi T/T_{\text{eff}}$  and the effective temperature is defined from  $k_B T_{\text{eff}} = \hbar\omega_c$ .

We are interested in the primary contribution to the low-temperature asymptotic behavior of the Casimir–Polder free energy (1) in the case  $\varepsilon_0 < \infty$  (i.e. with dc conductivity disregarded). For this purpose, as shown in [21–23], it is sufficient to restrict ourselves to a frequency-independent permittivity  $\varepsilon(i\xi_l) = \varepsilon_0$ . By expanding  $\Phi_A(x)$  in (2) in powers of  $x$  and using (3) and (4), one obtains

$$\Phi_A(x) = -\alpha_0 \left[ 4r_0 + r_0 \left( 4\beta_A^2 - 2\frac{\varepsilon_0}{\varepsilon_0 + 1} - 1 \right) x^2 + C_D(\varepsilon_0)x^3 \right], \tag{6}$$

where the terms in higher powers are omitted and the following notation is introduced:

$$C_D(\varepsilon_0) = r_0 \frac{7\varepsilon_0 + 1}{3(\varepsilon_0 + 1)} + \frac{(\sqrt{\varepsilon_0} - 1)[(3\varepsilon_0^2 + 1)(2\sqrt{\varepsilon_0} + 1) + 3\varepsilon_0(\sqrt{\varepsilon_0} - 1)]}{3(\sqrt{\varepsilon_0} + 1)(\varepsilon_0 + 1)^2} + \frac{2\varepsilon_0^2}{(\varepsilon_0 + 1)^{5/2}} \left( \text{Artanh} \sqrt{\frac{\varepsilon_0}{\varepsilon_0 + 1}} - \text{Arcoth} \sqrt{\varepsilon_0 + 1} \right). \tag{7}$$

In the limiting case  $\varepsilon_0 \rightarrow 1$  we have  $C_D(\varepsilon_0) \rightarrow 0$  as expected. The typical values of this coefficient are  $C_D(\varepsilon_0) = 0.585$  and  $7.60$  for  $\varepsilon_0 = 1.5$  and  $16$ , respectively. For the commonly used dielectrics such as  $\text{SiO}_2$  with  $\varepsilon_0 = 3.81$  and  $\text{Si}$  with  $\varepsilon_0 = 11.67$ , from (7) we get  $C_D(\varepsilon_0) = 2.70$  and  $6.33$ , respectively.

As a result, from (6) we obtain

$$\Phi_A(i\tau t) - \Phi_A(-i\tau t) = 2\tau^3 t^3 \alpha_0 C_D(\varepsilon_0). \tag{8}$$

Then from (5) the Casimir–Polder free energy is given by

$$\mathcal{F}(a, T) = E(a) - \frac{\hbar c \pi^3}{240a^4} \alpha_0 C_D(\varepsilon_0) \left( \frac{T}{T_{\text{eff}}} \right)^4 \tag{9}$$

and the Casimir–Polder entropy by

$$S(a, T) = -\frac{\partial \mathcal{F}(a, T)}{\partial T} = \frac{\pi^3 k_B}{30a^3} \alpha_0 C_D(\varepsilon_0) \left(\frac{T}{T_{\text{eff}}}\right)^3. \quad (10)$$

As can be seen from (10), the entropy goes to zero when  $T$  vanishes in accordance with the Nernst heat theorem. Thus, the Lifshitz theory of the atom–plate interaction is thermodynamically consistent if the dc conductivity of the dielectric plate is disregarded.

In electrodynamics the inclusion of the dc conductivity is equivalent to the replacement of  $\varepsilon(\omega)$  with

$$\tilde{\varepsilon}(\omega, T) = \varepsilon(\omega) + \frac{4\pi i \sigma_0(T)}{\omega}. \quad (11)$$

In the Lifshitz theory this leads to only negligible additions to all the terms at  $\omega = i\xi_l$  with  $l \geq 1$  in the free energy and entropy. These additions exponentially decay to zero with vanishing temperature [21–23]. However, the term with  $l = 0$  is modified because according to (3)  $r_{\text{TM}}(0, y) = r_0$  is replaced with  $\tilde{r}_{\text{TM}}(0, y) = 1$ . As a result, with dc conductivity included the free energy of the atom–plate interaction at low temperature is given by

$$\tilde{\mathcal{F}}(a, T) = \mathcal{F}(a, T) - \frac{k_B T}{4a^3} (1 - r_0) \alpha_0, \quad (12)$$

where  $\mathcal{F}(a, T)$  is defined in (9). From (12) one immediately arrives at the violation of the Nernst heat theorem

$$\tilde{S}(a, 0) = \frac{k_B \alpha_0}{4a^3} (1 - r_0) > 0. \quad (13)$$

### 3. Attempt to include the screening effects

Now we apply the above thermodynamic test to the generalization of the Lifshitz theory taking into account the screening effects. As is well known, such effects arise when a static electric field penetrates into a medium with a nonzero concentration of charge carriers. In this case, the reflection coefficient  $r_{\text{TM}}(0, y)$ , as given in (3) (the standard Lifshitz theory), is modified to

$$r_{\text{TM}}^{\text{mod}}(0, y) = \frac{\varepsilon_0 \sqrt{4a^2 \kappa^2 + y^2} - y}{\varepsilon_0 \sqrt{4a^2 \kappa^2 + y^2} + y}, \quad (14)$$

where  $\kappa^2 = 4\pi e^2 n / (\varepsilon_0 k_B T)$  and  $n$  is the total density of free charge carriers, while all the coefficients  $r_{\text{TM,TE}}(i\xi_l, y)$  with  $l \geq 1$  and also  $r_{\text{TE}}(0, y) = 0$  remain unchanged [25]. Here,  $\kappa$  is connected with the so-called *Debye radius*,  $\kappa = 1/R_D$ . When  $n = 0$ , (14) leads to the same result as (3). For  $n \rightarrow \infty$ , at fixed  $T \neq 0$ ,  $r_{\text{TM}}^{\text{mod}}(0, y) = 1$ , as in the case of the standard Lifshitz theory when the dc conductivity is included.

The calculation of the free energy at low temperature with the modified reflection coefficient (14) results in

$$\mathcal{F}^{\text{mod}}(a, T) = \mathcal{F}(a, T) - \frac{k_B T \alpha_0}{8a^3} \int_0^\infty r_{\text{TM}}^{\text{mod}}(0, y) e^{-y} y^2 dy + \frac{k_B T \alpha_0}{4a^3} r_0, \quad (15)$$

where  $\mathcal{F}(a, T)$  is defined in (9). The respective Casimir–Polder entropy is given by

$$\begin{aligned} S^{\text{mod}}(a, T) = S(a, T) + \frac{k_B \alpha_0}{4a^3} & \left[ \frac{1}{2} \int_0^\infty r_{\text{TM}}^{\text{mod}}(0, y) e^{-y} y^2 dy - r_0 \right] \\ & + \frac{k_B T \alpha_0}{8a^3} \int_0^\infty \frac{\partial r_{\text{TM}}^{\text{mod}}(0, y)}{\partial T} e^{-y} y^2 dy \end{aligned} \quad (16)$$

with  $S(a, T)$  defined in (10). It is easily seen that the last term on the right-hand side of (16) goes to zero when temperature vanishes, regardless of the temperature dependence of  $n$ . The second term on the right-hand side of (16) is more involved. If  $n(T)$  exponentially decays to zero with vanishing temperature (as is true for insulators and intrinsic semiconductors), then so does  $\kappa(T)$ . As a result,  $r_{\text{TM}}^{\text{mod}}(0, y) \rightarrow r_0$  and the entropy  $S^{\text{mod}}(a, 0)$  is equal to zero, in accordance with the Nernst theorem. However, if  $n$  does not go to zero when  $T$  goes to zero (this is true, for instance, for dielectric materials, such as semiconductors doped below critical and solids with ionic conductivity),  $\kappa \rightarrow \infty$  with vanishing temperature and  $r_{\text{TM}}^{\text{mod}}(0, y) \rightarrow 1$  when  $T \rightarrow 0$ . In this case we obtain from (16)

$$S^{\text{mod}}(a, 0) = \tilde{S}(a, 0) = \frac{k_B \alpha_0}{4a^3} (1 - r_0) > 0, \tag{17}$$

i.e. the proposed generalization violates the Nernst heat theorem in the same way as the standard Lifshitz theory with included dc conductivity (compare with (13)). In fact, conductivity  $\sigma_0(T) = n|e|\mu$ , where  $\mu$  is a mobility of charge carriers [27]. Although  $\sigma_0(T)$  goes to zero exponentially fast for all dielectrics when  $T$  goes to zero, for most of them this happens due to the vanishing mobility. For instance, the conductivity of SiO<sub>2</sub> (used in the calculations [25] and, as the plate material, in the experiment [7]) is ionic in nature and is determined by the concentration of impurities (alkali ions) which are always present as trace constituents. According to our result, for this material the generalization of the Lifshitz theory proposed in [25] violates the Nernst theorem.

The existence of dielectric materials for which  $n$  does not go to zero but  $\mu$  does go to zero when  $T$  vanishes demonstrates that the reflection coefficient (14) is ambiguous. In reality, for such materials  $r_{\text{TM}}^{\text{mod}}(0, y) \rightarrow 1$  when  $T$  and  $\mu$  simultaneously vanish. This is because  $\kappa \rightarrow \infty$  when  $T \rightarrow 0$  in disagreement with physical intuition that there should be no screening at zero mobility. In the conclusions, this ambiguity is linked to the break in continuity of the more general reflection coefficients at the point  $\omega = 0, T = 0$ .

The reflection coefficient (14) can be formally obtained from the standard Lifshitz theory with dissimilar permittivities  $\varepsilon_x = \varepsilon_y$  and  $\varepsilon_z$  commonly used for the description of uniaxial crystals [28, 29]. In this case the transverse-magnetic reflection coefficient at zero frequency is given by

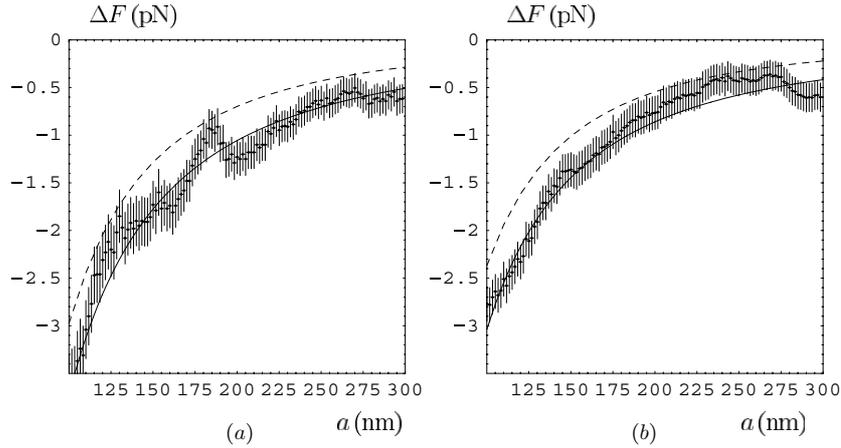
$$r_{\text{TM}}^{\text{uni}}(0, y) = \frac{\sqrt{\varepsilon_{0x}\varepsilon_{0z}} - 1}{\sqrt{\varepsilon_{0x}\varepsilon_{0z}} + 1}. \tag{18}$$

Then (14) follows from (18) if one puts

$$\varepsilon_{0x} = \varepsilon_0, \quad \varepsilon_{0z} = \varepsilon_0 \left( 1 + \frac{4a^2 \kappa^2}{y^2} \right). \tag{19}$$

However, an important difference of (18) and (19) from permittivities used in the case of uniaxial crystals is that (19) depends on the wave vector. This is in fact an extension of the Lifshitz theory with the inclusion of spatial dispersion. Such an inclusion is controversial and has been debated for long in the literature (see the negative conclusions regarding such an inclusion in [30] and the recent discussion [31]).

The question of whether there is a possibility of comparing the theoretical predictions of [25] with experimental data should be considered. This could be done with regard to the experiments on measuring the Casimir–Polder interaction between an atom and a SiO<sub>2</sub> plate [7] and the Casimir interaction between an Au-coated sphere and a Si plate [13–15]. In both cases the TE reflection coefficient at zero frequency does not contribute to the theoretical result. The experiment [7] was successfully used [24] to demonstrate that the inclusion of the dc conductivity of SiO<sub>2</sub> in the standard Lifshitz theory is inconsistent with the data. This is in



**Figure 1.** Difference of the Casimir forces between an Au-coated sphere and a Si plate in the presence and the absence of laser light on the plate versus separation ( $a$ ) for the absorbed power of 9.3 mW and (b) 4.7 mW. The experimental data are shown as crosses. Solid and dashed lines indicate the theoretical results calculated using the standard Lifshitz theory with the dc conductivity of Si in the dark phase neglected and the generalization of this theory [25], respectively.

support of our result in section 2 that such an inclusion leads to a contradiction between the Lifshitz theory and thermodynamics. In this connection the phenomenological prescription was formulated [32] that for dielectric materials the conductivity arising at nonzero temperature should be disregarded in the calculation of the Casimir force using the Lifshitz theory.

Recently one more experiment has been performed on measuring the difference of the Casimir forces between an Au sphere and B doped p-type Si plate illuminated with laser pulses [16, 17]. In the absence of laser pulse the concentration of charge carriers in a Si plate was  $\tilde{n} = 5 \times 10^{14} \text{cm}^{-3}$  and in the presence of pulse  $n_1 = 2.1 \times 10^{19} \text{cm}^{-3}$  or  $n_2 = 1.4 \times 10^{19} \text{cm}^{-3}$  for two different absorbed powers  $P_{w1} = 9.3 \text{ mW}$  and  $P_{w2} = 4.7 \text{ mW}$ . The directly measured quantity was  $\Delta F(a) = F^L(a) - F(a)$  where  $F^L$  and  $F$  are the Casimir forces in the presence and the absence of laser light on the plate, respectively. The experimental data were compared with the Lifshitz theory with neglected or included dc conductivity of the high-resistivity Si in the dark phase. In the latter case the theoretical model was found to be inconsistent with the data. Here, we compare the measurement data of this experiment (shown as crosses in figure 1 with experimental errors in force measurement determined at a 70% confidence level) with computations on the basis of the standard Lifshitz theory with the dc conductivity neglected in the dark phase (solid lines) and on the basis of its generalization taking into account the screening effects [25] (dashed lines). Figures 1(a) and (b) are related to the absorbed powers  $P_{w1}, P_{w2}$ , respectively. The experiment was performed in a high vacuum at  $T = 300 \text{ K}$  (see [17] for details). The solid lines were computed using the standard Lifshitz formula for the Casimir force with Si in the dark phase described as dielectric, i.e., by  $\varepsilon(i\xi_l)$  with  $\varepsilon(0) = \varepsilon_0 < \infty$ . In the presence of light the dielectric permittivity

$$\varepsilon^L(i\xi_l) = \varepsilon(i\xi_l) + \frac{\omega_{p(e)}^2}{\xi_l^2} + \frac{\omega_{p(p)}^2}{\xi_l^2} \quad (20)$$

has been used with the values of plasma frequencies for electrons and holes determined in [17] for different absorbed powers. Almost the same results are obtained if the Drude description of charge carriers is used in the presence of light

$$\tilde{\varepsilon}(i\xi_l) = \varepsilon(i\xi_l) + \frac{\omega_{p(e)}^2}{\xi_l[\xi_l + \gamma^{(e)}]} + \frac{\omega_{p(p)}^2}{\xi_l[\xi_l + \gamma^{(p)}]} \quad (21)$$

(see [17] for the values of all parameters at different absorbed powers). The dashed lines are obtained using equation (14) for the zero frequency TM reflection coefficient with different concentrations of charge carriers  $n = \tilde{n}$  in the dark phase and  $n = 2n_1$  or  $2n_2$  in the presence of light. At all nonzero Matsubara frequencies, in accordance with [25], the standard terms of the Lifshitz formula were used. The gold-coated sphere was described by the commonly used dielectric permittivity along the imaginary frequency axis (see, e.g., [12, 17, 18]). We have verified that for Au the use of expression (14) instead of the standard zero-frequency term, as given by the Lifshitz theory, leads numerically to the same results up to five significant figures. As is seen in figures 1(a) and (b), the experimental data are consistent with the theoretical results computed on the basis of the standard Lifshitz theory with the dc conductivity of dielectric Si neglected in the dark phase (the solid lines). The theoretical results computed on the basis of the generalized Lifshitz theory [25] with the modified TM reflection coefficient at zero frequency are excluded by data at a 70% confidence level. The same conclusion follows from the third data set obtained in [17] at  $P_{w3} = 8.5$  mW absorbed power.

#### 4. Conclusions and discussion

To conclude, we have found the low-temperature behavior of the Casimir–Polder free energy and entropy in the atom–plate configuration on the basis of the Lifshitz theory. For a dielectric plate with the dc conductivity of the plate material neglected, the Lifshitz theory was shown to be thermodynamically consistent. With the dc conductivity included, the standard Lifshitz theory of atom–plate interaction violates the Nernst heat theorem. The thermodynamic test was also applied to the recent generalization of the Lifshitz theory taking into account the screening effects. The proposed generalization is shown to be in violation of the Nernst theorem for a wide range of dielectric materials including doped semiconductors with doping concentration below critical and ionic conductors. This generalization is also inconsistent with the measurements of the difference Casimir force between a metal sphere and Si plate illuminated with laser pulses. Recently, the generalization of the Lifshitz theory [25] was extended using the Boltzmann transport equation [33]. This approach, in addition to the standard drift current  $\mathbf{j}$ , takes into account the diffusion current  $eD\nabla n$ , where  $D$  is the diffusion constant. In the quasi-static limit  $\omega \rightarrow 0$  the reflection coefficient (14) was reproduced. However, if one takes the limit  $T \rightarrow 0$  first, keeping  $\omega = \text{const} \neq 0$ , the standard Fresnel reflection coefficients (3) on the dielectric surface with no screening are obtained. This property is preserved in the subsequent limiting transition  $\omega \rightarrow 0$ . The calculation of the difference Casimir force in the experimental configuration [16, 17] using the reflection coefficients [33] defined at all Matsubara frequencies leads to precisely the same results, as shown by the dashed lines in figures 1(a) and (b). Thus, the theoretical approach [33] is also inconsistent with experiment. This is in line with the above phenomenological prescription stating that conductivity arising in dielectric materials at  $T \neq 0$  should be disregarded.

One may attempt to explain the physical reason why the Lifshitz theory does not allow the inclusion of real conductivity processes as follows. Lifshitz derived his famous formulae under the condition of thermal equilibrium. This means that not only  $T = \text{const}$ , but also all irreversible processes connected with the dissipation of energy into heat have already been terminated [34, 35]. The Drude-like dielectric function (11) is derived from the Maxwell equations with a real drift current of conduction electrons  $\mathbf{j} = \sigma_0 \mathbf{E}$  initiated by the external electric field  $\mathbf{E}$  [36]. The drift current is an irreversible process which brings a system out of

thermal equilibrium. The characteristic of this irreversible process, the Drude type of dielectric function, can also be derived within the Kubo formalism [37] using the correlation function of the electric field averaged over the thermodynamic state functions which are assumed to have the same values as they would have in an equilibrium situation. This allows a theoretical description of irreversible processes under the assumption that there is local equilibrium. The real current leads to Joule's heating of the Casimir plates (Ohmic losses) [38]. To preserve the temperature constant, one should admit that there exists an unidirectional flux of heat from the medium to the heat reservoir [39]. Such interactions between a system and a heat reservoir are prohibited by the definition of thermal equilibrium. Although the screening and diffusion effects really occur in an external electric field, they are also related to physical situations out of thermal equilibrium. The reason is that the diffusion current is determined by a nonzero gradient of charge carrier density, whereas for homogeneous systems in thermal equilibrium the charge carrier density must be homogeneous.

Our conclusion is that the standard Lifshitz theory of both atom-wall and wall-wall interactions is in a very good agreement with both thermodynamics and all available experiments. Violations of the Nernst theorem and contradictions with the experimental data arise when the theory is applied to physical phenomena involving electric current, be it of drift or diffusion type. Such an application seems to be in violation of thermal equilibrium which is the basic applicability condition of the Lifshitz theory.

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